TWO-DIMENSIONAL THERMAL ELASTICITY PROBLEM FOR A BODY WEAKENED BY A SYSTEM OF THERMALLY INSULATED CRACKS

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The thermally elastic state of a body in two dimensions with cracks has been investigated in a number of articles (see the survey in [1]). However, in the majority of cases problems have been investigated in which temperature stresses in a body are weakened by a single crack. The existing solutions of problems on the interaction between thermally insulated cracks in an elastic body have been confined to simple cases either with collinear [2, 3] or with arched cracks [4, 5]. Below the two-dimensional thermoelastic problem for an infinite body with arbitrarily positioned straight-lined thermally insulated cracks is studied by reducing it to a system of singular integral equations. An approximate solution is found for large distances between cracks. An exact solution is obtained in the case of a periodic system of collinear cracks.

1. A two-dimensional problem of heat conduction is considered for a body with thermally insulated cracks. Let there be N straight-line cuts (cracks) of length $2a_k\,(k=1, 2, \ldots, N)$ (see Fig. 1) in a plane with a Cartesian coordinate system xOy. At the centers $O_k\,(x_k^0, y_k^0)$ of the cracks there are positioned the origins $x_kO_ky_k$, of local coordinate systems whose O_kx_k axes coincide with the lines of the cracks each making an angle α_k with the Ox axis. It is assumed that in a continuous plane with no cuts the temperature distribution is described by a given harmonic function $t_0(x, y)$.

The determining of the stationary temperature field in a plane containing one crack, $|x_k|\leqslant a_k$, $y_k=0$, reduces to the solving of the following singular integral equation [6]:

$$\frac{1}{\pi}\int\limits_{-a_k}^{a_k}\frac{\gamma_k'(t)\,dt}{t-x_k}=-2\left.\frac{\partial t_0(x, y)}{\partial y_k}\right|_{y_k=0}=f_k(x_k),\quad |x_k|\leqslant a_k, \tag{1.1}$$

where $\gamma_k(x_k)=0.5\,[t_k^+(x_k, 0)-t_k^-(x_k, 0)]$ is the density of the Cauchy integral

$$F_k(z_k)=\frac{1}{2\pi i}\int\limits_{-a_k}^{a_k}\frac{\gamma_k(t)\,dt}{t-z_k},\; z_k=x_k+iy_k, \tag{1.2}$$

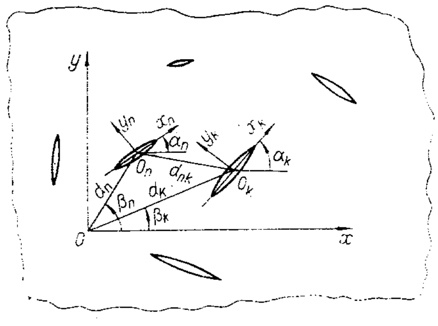

Fig. 1

which expresses the perturbed temperature field $t_k(x_k, y_k)$, due to the crack $t_k(x_k, y_k)=\mathrm{Re}\,F_k(z_k)$. The total temperature field in such a domain is equal to $T_k(x, y)=t_k(x_k, y_k)+t_0(x, y)$.

The solution of Eq. (1.1) which is unbounded at both ends of the interval $[-a_k, a_k]$ is given by [7].

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$$\gamma_k'(x) = -\frac{1}{\pi\sqrt{a_k^2 - x^2}} \int_{-a_k}^{a_k} \frac{\sqrt{a_k^2 - t^2}\, f_k(t)\, dt}{t - x}. \tag{1.3}$$

It can be shown that the function $F_k(z_k)$ is invariant with respect to a parallel translation of the coordinate axes though it is not invariant with respect to the rotation of the coordinate axes. If the coordinate system $x_kO_ky_k$ is related to the xOy system by the relation

$$z = z_k e^{i\alpha_k} + z_k^0, \qquad z_k^0 = x_k^0 + iy_k^0,$$

and the function F(z) plays the same part in the xOy system as the function $F_k(z_k)$ in the $x_kO_ky_k$ system, then

$$F_k(z_k) = e^{i\alpha_k} F\left(z_k e^{i\alpha_k} + z_k^0\right). \tag{1.4}$$

If on the N intervals $|x_k| \leqslant a_k$, $y_k = 0 (k = 1, 2, \ldots, N)$ the gaps $\gamma_k(x_k)$ in the temperature field are given, then the temperature distribution in the entire domain is given by the formula

$$T(x, y) = t(x, y) + t_0(x, y), \; t(x, y) = \mathrm{Re} F(z).$$

In the above the function

$$F(z) = \frac{1}{2\pi i} \sum_{k=1}^{N} e^{-i\alpha_k} \int_{-a_k}^{a_k} \frac{\gamma_k(t)\, dt}{t - z_k}$$

is obtained by the superposition of the functions $F_k(z_k)$ of (1.2) for single cracks and by taking into account the transformation formula (1.4) when changing to another coordinate system.

If the boundary conditions

$$\left.\frac{\partial T}{\partial y_k}\right|_{y_k=0} = 0, \; |x_k| \leqslant a_k \; (k = 1, 2, \ldots, N),$$

are satisfied on the boundaries of the cracks, then to determine the unknown functions $\gamma_k(x_k)$ one obtains a system of singular integral equations,

$$\int_{-a_n}^{a_n} \frac{\gamma_n'(t)\, dt}{t - x} + {\sum_{k=1}^{N}}' \int_{-a_k}^{a_k} \gamma_k'(t) R_{nk}(t, x)\, dt = \pi f_n(x), \; |x| \leqslant a_n \; (n = 1, 2, \ldots, N). \tag{1.5}$$

The notation $\sum'$ indicates that the term with the row number should be omitted in the summation. The kernels $R_{nk}(t, x)$ are found by using the relations

$$R_{nk}(t, x) = \mathrm{Re}\left(\frac{e^{i\alpha_k}}{T_k - X_n}\right), \; T_k = te^{i\alpha_k} + z_k^0, \; X_n = xe^{i\alpha_n} + z_n^0.$$

Thus the determination of the stationary temperature field in a plane with thermally insulated cuts has been reduced to the solving of a system of singular integral equations (1.5). It is noted that in [8] Eqs.(1.5)were obtained in a somewhat different way.

The solution is now found in the case of large distances between cracks. Then for the kernels $R_{nk}(t,x)$ the expansions

$$R_{nk}(t, x) = \sum_{p=0}^{\infty} \sum_{\nu=0}^{p} c_{nkp\nu} t^{\nu} x^{p-\nu} d_{nk}^{-p-\nu},$$

are valid; here

$$c_{nkp\nu} = (-1)^{p+\nu+1} C_p^\nu \cos[(p-\nu+1)\alpha_n + \nu\alpha_k - (p+1)\beta_{nk}];$$

$$C_p^\nu = \frac{p(p-1)\ldots(p-\nu+1)}{\nu!}; \quad d_{nk}e^{i\beta_{nk}} = z_n^0 - z_k^0.$$

By introducing the dimensionless parameter

$$\lambda = \frac{2a}{d}, \quad a = \max\{a_n\}, \quad d = \min\{d_{nk}\},$$

which characterizes the distances between cracks the solution of the system (1.5) of integral equations is found, following [9], as the series

$$\gamma_n'(x) = \sum_{p=0}^{\infty} \gamma_{np}'(x)\lambda^p,$$

where

$$\gamma_{n0}'(x) = -\frac{1}{\pi\sqrt{a_n^2 - x^2}} \int_{-a_n}^{a_n} \frac{\sqrt{a_n^2 - t^2}\, f_n(t)\,dt}{t - x}, \quad \gamma_{n1}'(x) = 0;$$

$$\gamma_{np}'(x) = \frac{1}{\pi\sqrt{a_n^2 - x^2}} {\sum_{k=1}^{N}}' \sum_{s=1}^{p-1} \sum_{\nu=1}^{s} H_{s-\nu}\left(\frac{x}{a_n}\right)\left(\frac{\varepsilon_{kn}}{2}\right)^{s+1} a_n^{-\nu} a_{nks\nu} \int_{-a_k}^{a_k} t^\nu \gamma_{k,p-s-1}'(t)\,dt \quad (p = 2, 3, \ldots);$$

$$H_p\left(\frac{x}{a_n}\right) = \frac{1}{\pi a_n^{p+1}} \int_{-a_n}^{a_n} \frac{\xi^p \sqrt{a_n^2 - \xi^2}}{\xi - x}\,d\xi, \quad \varepsilon_{nk} = \frac{a_k d}{a d_{nk}} \leqslant 1.$$

For collinear cracks the system (1.5) has a solution in closed form.

2. When solving the thermal elasticity problem for a plane with cuts one has to determine the thermo-elastic state of the continuous plane due to the temperature field $t_0(x, y)$, to find the stress components on the cuts, and then to solve the force problem [9]; the latter is done by taking into account that forces have been applied to the boundaries of the cuts which are equal in magnitude but opposite in sign to the found stresses. Moreover, the stress distribution due to the perturbed temperature field $t(x,y)$ must also be established. This will be considered later when the solutions to the first two problems have been found.

It is assumed now that there is one crack $|x_k| \leqslant a_k$, $y_k = 0$, in an elastic isotropic plane whose edges do not touch and are not loaded. Then

$$N_k^+ - iT_k^+ = N_k^- - iT_k^- = 0, \quad |x_k| \leqslant a_k, \tag{2.1}$$

where N_k, T_k represent the normal and tangential stresses, respectively, on the O_k, x_k axis.

The following notation is now introduced:

$$u_k^+ - u_k^- + i(v_k^+ - v_k^-) = \frac{i(\varkappa+1)}{2G} g_k(x_k), \quad |x_k| \leqslant a_k. \tag{2.2}$$

By using the formulas [10, 11]

$$N_k - iT_k = \Phi_k(z_k) + \overline{\Phi_k(z_k)} + z_k\overline{\Phi_k'(z_k)} + \overline{\Psi_k(z_k)};$$

$$2G\frac{\partial}{\partial x_k}(u_k + iv_k) = \varkappa\Phi_k(z_k) - \overline{\Phi_k(z_k)} - z_k\overline{\Phi_k'(z_k)} - \overline{\Psi_k(z_k)} + \beta F_k(z_k),$$

one obtains from the conditions (2.1) and (2.2) the conjugate problem on the interval $|x_k| \leqslant a_k$ for the function $\Phi_k(z_k)$,

$$\Phi_k^+(x_k) - \Phi_k^-(x_k) = iG_k(x_k),\ G_k(x) = g_k'(x) + \frac{i\beta\gamma_k(t)}{\varkappa+1}, \tag{2.3}$$

where $\Phi_k(z_k)$, $\Psi_k(z_k)$ are complex stress potentials [10]; $F_k(z_k)$ is given by the formula (1.2); $\varkappa=3-4\nu$, $\beta=\alpha E$ for a deformation in a plane and $\varkappa=(3-\nu)(1+\nu)$, $\beta=\alpha E\,(1+\nu)$ for a generalized two-dimensional stressed state*; α is the temperature expansion coefficient; $G=E/2(1+\nu)$ is the shear modulus, E is the elasticity modulus; ν is the Poisson coefficient.

From the boundary-value problem (2.3) the function $\Phi_k(z_k)$ which decreases at infinity is given by the Cauchy integral [10]

$$\Phi_k(z_k) = \frac{1}{2\pi}\int_{-a_k}^{a_k}\frac{G_k(t)dt}{t-z_k}.$$

Hence one can draw the conclusion that the integral equations of the thermoelastic problem for a body with cracks are identical with those for the corresponding force problem [9, 12] provided that in the latter the unknown functions $g_k'(x)$ are replaced by $G_k(x)$. In the case of a single straight line crack $|x_k| \leqslant a_k$, $y_k=0$, whose edges are not loaded one has the equation

$$\int_{-a_k}^{a_k}\frac{G_k(t)\,dt}{t-x} = 0,\ |x| \leqslant a_k,$$

and its solution is given by [7]

$$G_k(x) = \frac{iA_k}{\sqrt{a_k^2 - x^2}}. \tag{2.4}$$

Integrating the latter from $-a_k$ to a_k, one finds the value of the constant A_k.

$$A_k = \frac{\pi\beta}{\varkappa+1}\int_{-a_k}^{a_k}\gamma_k(x)\,dx = -\frac{\pi\beta}{\varkappa+1}\int_{-a_k}^{a_k}x\gamma_k'(x)\,dx. \tag{2.5}$$

In the above one has taken into account that $\gamma_k(-a_k)=\gamma_k(a_k)=0$. By substituting $\gamma_k(x)$ from (1.3) into (2.5) one obtains

$$A_k = \frac{\pi\beta}{\varkappa+1}\int_{-a_k}^{a_k}\sqrt{a_k^2 - t^2}\, f_k(t)\,dt.$$

If the function $G_k(x)$ is known, one is able to determine the thermoelastic state over the entire plane. In particular, the coefficients of stress intensity at the vertices of the crack are found from the formula

$$k_{1k}^{\pm} - ik_{2k}^{\pm} = \mp\lim_{x_k\to\pm a_k}\left[\frac{\sqrt{a_k^2 - x_k^2}}{\sqrt{a_k}}G_k(x_k)\right]. \tag{2.6}$$

In the above the upper sign refers to the right vertices of the crack, and the lower one to the left vertices. By using the relations (2.4) and (2.6) one can write

*In this case it has been assumed that the plate is thermally insulated on its lateral surfaces.

$$k_{2k}^{\pm} = \mp \frac{\beta}{\pi(\varkappa+1)\sqrt{a_k}} \int_{-a_k}^{a_k} f_k(t) \sqrt{a_k^2 - t^2}\, dt, \quad k_{1k}^{\pm} = 0. \tag{2.7}$$

For a homogeneous heat flow at infinity which is perpendicular to the Ox axis, that is, for

$$t_0(x, y) = qy, \tag{2.8}$$

one finds from the formulas (2.7) and (2.8) that

$$k_{2k}^{\pm} = \pm \frac{\beta q a_k \sqrt{a_k} \cos\alpha_k}{\varkappa+1}, \quad k_{1k}^{\pm} = 0. \tag{2.9}$$

This result was previously obtained in [13].

In the case of a system of cracks $|x_k| \leqslant a_k$, $y_\varkappa = 0\,(k = 1, 2, \ldots, N)$, directed arbitrarily and free of loads the integral equations are given by [9]

$$\int_{-a_n}^{a_n} \frac{G_n(t)\, dt}{t-x} + \sum_{k=1}^{N}{}' \int_{-a_k}^{a_k} \left[G_k(t) K_{nk}(t, x) + \overline{G_k(t)} L_{nk}(t, x)\right] dt = 0, \quad |x| \leqslant a_n\, (n = 1, 2, \ldots, N). \tag{2.10}$$

In the above

$$K_{nk}(t, x) = \frac{e^{-i\alpha_k}}{2} \left(\frac{1}{T_k - X_n} + \frac{e^{-2i\alpha_n}}{\overline{T}_k - \overline{X}_n} \right);$$

$$L_{nk}(t, x) = \frac{e^{-i\alpha_k}}{2} \left[\frac{1}{\overline{T}_k - \overline{X}_n} - \frac{T_k - X_n}{(\overline{T}_k - \overline{X}_n)^2} e^{-2i\alpha_n} \right].$$

By using the inversion formula for Cauchy integrals [7], a system of Fredholm integral equations of the second kind is found from the formula (2.10),

$$G_n(x) = \frac{1}{\pi\sqrt{a_n^2 - t^2}} \left\{ iA_n + \sum_{k=1}^{N}{}' \int_{-a_k}^{a_k} \left[G_k(t) M_{nk}(t, x) + \overline{G_k(t)} N_{nk}(t, x)\right] dt \right\}, \quad |x| \leqslant a_n\, (n = 1, 2, \ldots, N). \tag{2.11}$$

In the above one has

$$\begin{pmatrix} M_{nk}(t, x) \\ N_{nk}(t, x) \end{pmatrix} = \frac{1}{\pi} \int_{-a_n}^{a_n} \frac{\sqrt{a_n^2 - \xi^2}}{\xi - x} \begin{pmatrix} K_{nk}(t, \xi) \\ L_{nk}(t, \xi) \end{pmatrix} d\xi.$$

For large distances between cracks the solution of the system (2-11) can be obtained in the series form [9]

$$G_n(x) = \sum_{p=0}^{\infty} G_{np}(x) \lambda^p,$$

where

$$G_{no}(x) = \frac{iA_{n0}}{\pi\sqrt{a_n^2 - x^2}}, \quad G_{n1}(x) = 0;$$

$$G_{np}(x)=\frac{1}{\pi\sqrt{a_n^2-x^2}}\left\{\sum_{k=1}^{N'}\sum_{s=1}^{p-1}\sum_{\nu=1}^{s}H_{s-\nu}\left(\frac{x}{a_n}\right)\left(\frac{\varepsilon_{kn}}{2}\right)^{s+1}\times\right.$$

$$\left.\times a_n^{-\nu}\int_{-a_k}^{a_k}t^{\nu}\left[a_{nks\nu}G_{k,p-s-1}(t)+b_{nks\nu}\overline{G_{k,p-s-1}(t)}\right]dt+iA_{np}\right\}$$

$$(p=2,3\ldots).$$

In the above the values $a_{nkp\nu}$, $b_{nkp\nu}$, and A_{np} are coefficients of the expansion of the functions $K_{nk}(t, x)$, $L_{nk}(t, x)$, and A_n for small values of the parameter λ.

$$\begin{pmatrix}K_{nk}(t,x)\\ L_{nk}(t,x)\end{pmatrix}=\sum_{p=0}^{\infty}\sum_{\nu=0}^{p}\begin{pmatrix}a_{nkp\nu}\\ b_{nkp\nu}\end{pmatrix}t^{\nu}x^{p-\nu}d_{nk}^{\;-p-1};$$

$$A_n=\sum_{p=0}^{\infty}A_{np}\lambda^p;\ A_{np}=\frac{\beta}{\varkappa+1}\int_{-a_n}^{a_n}\gamma_{np}(x)\,dx;$$

$$a_{nkp\nu}=\frac{1}{2}(-1)^{p+\nu+1}C_p^{\nu}\{\exp\{i[(p+1)\beta_{nk}-(p-\nu)\alpha_n-(\nu+$$

$$+1)\alpha_k]\}+\exp\{i[(p+1)\beta_{nk}-(p-\nu+2)\alpha_n-(\nu-1)\alpha_k]\}\};$$

$$b_{nkp\nu}=\frac{1}{2}(-1)^{p+\nu}\{(p+1)C_p^{\nu}\exp\{i[(p+3)\beta_{nk}-(p-\nu+$$

$$+2)\alpha_n-(\nu+1)\alpha_k]\}-(C_p^{\nu}+pC_{p-1}^{\nu})\exp\{i[(p+1)\beta_{nk}-$$

$$-(p-\nu)\alpha_n-(\nu+1)\alpha_k]\}-pC_{p-1}^{\nu-1}\exp\{i[(p+1)\beta_{nk}-(p-\nu+2)\alpha_n-(\nu-1)\alpha_k]\}\}.$$

It should be noted that the approach used here to solve two-dimensional thermoelastic problems for bodies with cracks can also be employed in practical applications, since the problems of heat conduction and thermal elasticity, as well as the force problem, are all solved in the same manner. By employing this method one is able to obtain exact solutions to thermoelastic problems in the case of two collinear cracks of equal length, as well as for a periodic system of collinear cracks in the infinite plane. The former problem was analyzed in [2] and its solution is given below.

3. Let us consider a periodic system of collinear thermally insulated cracks located on the Ox axis ($\alpha_n=0, a_n=a, z_n^0=nd$, $n=0,\pm1,\pm2,\ldots$) and subjected to the same conditions ($f_n(x)=f(x)$). In this case the system (1.5) is reduced to a single integral equation ($\gamma_n(x)=\gamma_1(x)$) [12], namely,

$$\frac{1}{d}\int_{-a}^{a}\gamma_1'(t)\,\mathrm{ctg}\,\frac{\pi(t-x)}{d}\,dt=f(x),\ |x|\leqslant a.$$

Its solution is given by

$$\gamma_1'(x)=\frac{1}{d\sqrt{\mathrm{tg}^2\frac{\pi a}{d}-\mathrm{tg}^2\frac{\pi x}{d}}}\left[\frac{1}{\cos^2\frac{\pi x}{d}}\int_{-a}^{a}\frac{\sqrt{\mathrm{tg}^2\frac{\pi a}{d}-\mathrm{tg}^2\frac{\pi t}{d}}\,f(t)dt}{\mathrm{tg}\frac{\pi x}{d}-\mathrm{tg}\frac{\pi t}{d}}+C_2\right].\qquad(3.1)$$

Integrating (3.1) between the limits $-a$ to a one finds that $C_2 = 0$.

To the system (2.10) of integral equations there corresponds in this case ($G_n(x)=G_1(x)$) the equation

$$\int_{-a}^{a}G_1(t)\,\mathrm{ctg}\,\frac{\pi(t-x)}{d}\,dt=0,\ |x|\leqslant a,$$

whose solution is

$$G_1(x) = \frac{iC_3}{\sqrt{\operatorname{tg}^2 \frac{\pi a}{d} - \operatorname{tg}^2 \frac{\pi x}{d}}}.$$

By using the relation

$$\int_{-a}^{a} G_1(x)\,dx = iA_1,$$

one finds the value of the constant C_3:

$$C_3 = \frac{A_1}{d \cos \frac{\pi a}{d}}.$$

The solution has thus been reduced to the finding of the constant A_1. Substituting in (2.5) the value of $\gamma_1'(x)$ as given by (3.1) and inverting the order of integration and evaluating the internal integral, one obtains

$$A_1 = -\frac{\beta d}{2\pi(\varkappa+1)} \int_{-a}^{a} f(t) H(t)\,dt,$$

$$H(t) = \ln \left| \frac{\sqrt{1 + \operatorname{tg}^2 \frac{\pi a}{d}} + \sqrt{\operatorname{tg}^2 \frac{\pi a}{d} - \operatorname{tg}^2 \frac{\pi t}{d}}}{\sqrt{1 + \operatorname{tg}^2 \frac{\pi a}{d}} - \sqrt{\operatorname{tg}^2 \frac{\pi a}{d} - \operatorname{tg}^2 \frac{\pi t}{d}}} \right|.$$

The coefficients of stress intensity are found by using the formula (2.6). This produces

$$k_2^{\pm} = \mp \frac{\beta d}{2\pi(\varkappa+1)\sqrt{\pi d \operatorname{tg} \frac{\pi a}{d}}} \int_{-a}^{a} f(t) H(t)\,dt, \quad k_1^{\pm} = 0. \tag{3.2}$$

In the case of homogeneous heat flow at infinity $(f(x) = -2q)$ one finds from (3.2) that

$$k_2^{\pm} = \mp \frac{2\beta q d^2}{\pi(\varkappa+1)\sqrt{\pi d \operatorname{tg} \frac{\pi a}{d}}} \ln \left| \cos \frac{\pi a}{d} \right|, \quad k_1^{\pm} = 0. \tag{3.3}$$

One should mention in conclusion that the formulas (2.7) and (3.2) provide only those components of the coefficients of stress intensities which are due to perturbations of the temperature field. The results (2.9) and (3.3) represent a complete solution of the problem, since the linear temperature field $t_0(x, y)$ of (2.8) does not give rise to any stresses in the continuous infinite plane.

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